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Chapter 6

MENTAL REPRESENTATIONS OF STATISTICAL INFORMATION*

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ABSTRACT

This chapter reviews ongoing research on human statistical reasoning, looking at the relationships between how information is presented, how numerical information is subsequently represented in the mind, and how the resulting judgments are made. At each of these stages there are both emerging conclusions and ongoing debates. Information presented in the form of frequencies appears to be more easily accepted into the cognitive judgment mechanisms, as is information presented in clearly organized relationships. Clarifications are necessary, however, regarding what constitute frequency presentations and the isomorphism of organizational relationships that have been proposed to be facilitatory. The actual reasoning processes are constrained (or enabled, as it were) by the nature of these mental representations. Finally, the benchmarks by which human judgments are evaluated – the markers for claiming that humans are "good" or "poor" at statistical reasoning – are assessed.

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INTRODUCTION

It is a truism that the more similar two groups are, the more rancorous their resulting conflicts tend to be. This insight can be applied to the relationship between two major perspectives now in the field of judgment and decision making, whose proposals typically cover the same empirical landscape, resulting in territorial conflict often insulated by a minefield of rhetoric and heated debate. The perspective of *ecological rationality* – sometimes referred to as the "optimists"—issue claims of strong human capabilities (e.g. "frequentist problems elicit bayesian reasoning"— Cosmides & Tooby, 1996, p. 62) in areas for which human abilities were previously claimed to be meager or non-existent from the traditional *heuristics and biases* perspective – sometimes referred to as the "pessimists" (e.g., Matlin, 1998). These optimistic claims are attacked by pessimistic critics (e.g., "with probabilistic reasoning, and especially with reasoning about frequency probabilities, Cosmides and Tooby's results have very little to do at all..." – Howson & Urbach, 1993, p. 422), who are in turn rebutted by advocates of the optimists (e.g., "…rather than being superficial or mechanistic, natural frequencies can foster statistical insight."— p. 429, Gigerenzer & Hoffrage, 1999).

The smoke and rhetoric from this debate, however, should not detract from the fact that significant points of agreement are emergent between the pessimists and the optimists. Specifically:

1. As an overall evaluation across all real-world contexts, people are generally competent at making judgments and decisions (as the optimists claim, and the pessimists concede).
2. Despite this overall competence, people do sometimes make mistakes in judgments and decisions (as the pessimists claim and the optimists concede). The patterns of these mistakes can be used to help understand the cognitive processes underlying judgments and decisions in general.
3. Presenting numerical information in certain frequency formats tends to help people understand the information (even though there may be some disagreement as to why this is the case).
4. Putting information into natural sampling relationships (also called nested-set relations, partitive formulations, and variations of these labels) tends to help people understand the information.

The meaning and the significance of these points of agreement provide the grist for much of the debate between the two perspectives, but nevertheless these are points from which consensus and conciliation – and ultimately scientific progress – are most likely. First, however, this section will look at the two perspectives in general, because the heuristics and biases (pessimists) perspective and the ecological rationality (optimists) perspective actually invoke very different views about the nature of the human mind in general.

THE HEURISTICS AND BIASES MIND

The heuristics and biases tradition views the mind as a fairly general-purpose information processing machine, which has a limited processing capacity. So long as the demands of a judgment or decision do not exceed these limits, performance is typically accurate, conforming to normative models of human rationality. When information from the world overwhelms the mind's capacities, however, it falls back upon a set of general short-cut strategies –heuristics—for making judgments and decisions. These short-cut strategies are characterized by a range of errors and biases (e.g., the base rate fallacy), and therefore lead to judgments at variance with normative models of human judgment and decision making (Kahneman, Slovic, & Tversky, 1982).

The primary impetus for the heuristics and biases perspective was a reaction to the "Homo Economicus" model of human judgment and decision making that prevails in economics – many traditional models of economic systems assume that people always perform rationally and based on a completely accurate representation of the environmental situation. Research by the heuristics and biases pessimists has shown that this assumption is regularly violated in the real world. Humans are not perfectly accurate and rational; they are prone to various errors that are endemic enough to warrant the charge of irrationality. The heuristics and biases approach traditionally lays the blame for this situation on computational processing limitations of the mind.

THE ECOLOGICAL RATIONALITY MIND

The ecological rationality approach grew in large part out of a reaction to the heuristics and biases view (Gigerenzer, 1991), supplemented by greater attention to considerations of bounded rationality and evolutionary biology. It views the mind as a more diverse collection of abilities, each of which is designed by natural selection to deal with relatively specific aspects of the world (i.e., cognitive abilities are domain-specific). Furthermore, the specific domains in which the mind is designed to work can be inferred from both the enduring regularities in the structure of the world as it is now and the structure of the world in the evolutionary environment in which humans evolved. In short, this view is optimistic that humans are accurate and rational if one uses the criteria of judgments and decisions for which the mind is designed. Unlike the heuristics and biases approach, computational limitations are not necessarily the key stumbling block for human rationality but instead may enable important adaptive functions that support accurate judgments and decisions (e.g., the recognition heuristic; Hertwig & Todd, 2003). Thus, the ecological rationality approach attributes the observed pattern of errors and biases to a failure by researchers to test behavior under ecologically relevant conditions—that is, on placing people in situations (including providing types of information) that fall outside of those for which the mind was designed to function well (Brase, Cosmides, & Tooby, 1998; Gigerenzer, Todd, & The ABC Research Group, 1999).

REPRESENTING AND USING INFORMATION, STEP BY STEP

Many disagreements between the heuristics and biases pessimists and the ecological rationality optimists are reflections of these different underlying views of the nature of the human mind. A step-by-step review of how judgments and decisions are made can point out where these disagreements are, what they are, and how they are related to these different models of the mind. This chapter will therefore compare and contrast these two views regarding: a) the nature of statistical information in the environment, b) the nature of mental representations of numerical information, c) the processing of numerical information, and d) the evaluation of resulting judgments and decisions as competent or incompetent.

Statistical Information in the Environment

Both the heuristics and biases approach and the ecological rationality approach often look to the field of perception for inspiration and guidance. The heuristics and biases approach stresses the fact that human perception – while usually quite good – is prone to systematic errors (such as visual illusions), and that we might by analogy expect the same to be true of judgments and decision making processes. While this is true and – as noted above – consonant with both views, very few researchers would characterize perception as very irrational or based on limited processing abilities (since tasks such as the stereoptic recovery of depth information from disparate visual images are computationally quite complex).

The ecological rationality approach looks to the field of perception for guidance about what information is available in the environment to be used in judgments and decision making, and how information is selected from the environment. Like visual attention, which actually registers only a small fraction of the actual visual array to which one is exposed, ecological rationality advocates point out that only a small amount of the statistical information that could be obtained from the environment is likely to actually be registered by the mind. We have selective statistical attention to avoid wasting time with details of the environment that were not correlated in any way with survival or reproduction over human evolutionary history. So I am, at this moment, ignoring the number of pattern repeats in the carpet of this room, the number of time "the" has appeared on this page, the number of times the air conditioner has turned on and off, and the exact number of papers on my desk. Our minds are also not designed to pay attention to types of information and aspects of the world that were not available over evolutionary history. For example, it is often difficult to accurately gauge the true ramifications, on one's own life, of events about which we know but that are distant in time or space (e.g., violent crimes in other parts of the country, the fact that earth's magnetic poles will eventually shift and cause all sorts of chaos, and people will starve to death in another part of the world next year). The optimists emphasize that human judgment, like perception, is selectively focused on the most relevant details of the environment and produces results that are evolutionarily selected as the best options from those available.

What implications do these differing appeals to perception have on the resulting theory of judgment and decision making? Consider the following result: When asked which is more likely, (a) that you will die of cancer in the next 25 years, or (b) that you will die of heart disease in the next 25 years, many people choose option (a) as more likely (Tversky & Kahneman, 1973). Actuarial data, in fact, show that (b) is much more likely that (a), so this response is incorrect. The heuristics and biases approach labels this as the availability heuristic: cancer is more available in memory than heart disease (quite probably due to more media coverage), and is therefore assumed to be more prevalent. The jump from memory availability to prevalence is the error, and this inference is assumed to have been made because the information on actual prevalence was either unavailable or beyond the processing capabilities of the mind.

On the other hand, the ecological rationality approach notes that modern media coverage – and its biases towards certain topics over others (e.g., cancer rather than heart disease) – is not something that the mind evolved to take into account (i.e., there was no mass media over the first 99.9% of human evolutionary history). In the absence of modern media, one's information about factors that cause death would be experiences regarding the people in one's local region. This would not include high rates of sensational homicides, airline fatalities, or geographically foreign diseases; instead you would have a fairly accurate accounting of what people tend to actually die from in your area. This situation is analogous to visual illusions not only with respect to producing an inaccurate perception, but also with respect to being constrained to an artificial setting (visual illusions, such as the Necker cube, only work when they are presented as two-dimensional images that our minds struggle to make into three-dimensional representations). In short, the ecological rationality view asserts that the observed judgmental errors derive from a bias that exists in the environmental information itself, not in the mental ability, which was simply fed the biased information.

In other situations, information that is needed to make an accurate judgment is in the environment, but people do not seem to use it. For example, when people are asked to estimate the number of English words with "n" as the second to last letter (e.g., -----n-), they give lower estimates than when they are asked to estimate the number of English words with "ing" as the last three letters (e.g., ----ing; Tversky & Kahneman, 1973). Obviously, the latter is a subset of the former, and therefore, according to the extensional rule of probability theory, there must be more -----n- words than -----ing words.

Again, the heuristics and biases account cites the availability heuristic; words ending in –ing are more easily recalled, and therefore judged to be more common, than words with –n- as the second to last letter. The process of actually going through one's entire lexicon would presumably be too computationally demanding. The ecological rationality approach points to the fact that there is no reason to suppose that evolution designed the mind to track the frequency of things such as ----n- words. There is not even much in the way of a rationale for tracking ----ing words, but as this suffix is actually a unit of linguistic meaning, it is slightly easier to access the frequency with which it is used (Sedlmeier, Hertwig, & Gigerenzer, 1998; Gigerenzer & Goldstein, 1996).

Furthermore, there exist situations in which the processing limitations of the mind enable important adaptive functions that support accurate judgments and decisions, rather than leading to systematic errors and biases (e.g., Hertwig & Todd, 2003). For example, when people are asked to select the best university among a list of schools, decisions based purely on the recognition of each school can yield surprisingly accurate judgments that exceed performance by individuals who know more information about each school. Accurate decisions based on one's recognition of each university occurs when their exposure to each school is positively correlated with the school's ranking along the relevant decision criterion (e.g., which school is more academically selective), permitting the use of a simple "recognition heuristic" that supports accurate decisions (Goldstein & Gigerenzer, 1999, 2002). In contrast, knowing more information about each school may lead to a fairly complex decision process requiring the integration of multiple dimensions (e.g., academic selectivity rating, tuition expense, student:teacher ratio), resulting in poorer performance. Thus, when viewed from an ecological rationality perspective, the processing limitations of the mind may enable simple heuristics that support accurate judgments and decisions—exceeding, in some cases, the performance of more complex, normative models, suggesting that the processing limitations of the mind should not be viewed solely as a source of judgmental error and biases, but instead as an evolutionarily adaptive constraint that enables accurate decisions under ecologically relevant conditions.

Ecological rationality advocates also make a separate, but parallel, argument about the nature of statistical information in the environment. Although our modern environment is replete with statistical information expressed as single-events (e.g., a 60% chance of rain today), well over 99% of human evolutionary history did not include such numerical expressions. Over evolutionary history, information about the environment came largely from first-hand experience (even information provided by others would have been restricted to fairly recent events and those people with whom one actually lives). Experientially, single events either happen or they don't— either it will rain today or it will not. One can observe that it rained on 6 out of the last 10 days with cold winds and dark clouds, but one cannot observe a 60% subjective confidence (this is a separate claim from the assertion that one can *have* a cognitive experience of subjective confidence, which is entirely possible). As individuals have, over evolutionary history, been able to observe the frequency with which events occur, this information was potentially available to be utilized to improve decision making. Thus, if humans have adaptations for inductive reasoning, one might expect them to include procedures designed to take advantage of the existent frequency information in the environment (see Hasher & Zacks, 1979; Hintzman & Stern, 1978). Indications that the mind is designed to process natural frequencies is consistent with the ecological rationality proposal that the mind embodies inference procedures based on such information, but it is not sufficient for showing that such inference procedures in fact exist; only that the necessary information –in the ecologically relevant format—is available. On the other hand, given the lack of single-event probability information in the environment in which humans evolved and the relative complexity of processing these formats, there is no reason to expect natural selection to have built cognitive mechanisms well-designed to deal with information in a single-event probability format (Brase, et al., 1998).

In summary, both the heuristics and biases view and the ecological rationality view agree that there is vastly more statistical information extractable from the environment than is actually registered or used by the human mind. In one sense, the difference between these views at this level has to do with the placement of the information bottleneck. The heuristics and biases view asserts that the mind tries, and fails, to absorb all the relevant information in the environment (or at least a much larger quantity than actually possible). Habitually failing or unable to accomplish this task, judgment processes fall back on heuristics and biases. The ecological rationality view asserts that the mind *a priori* is focused on the most relevant aspects of the environment (and the statistical information embodied therein) for evolutionarily important judgments and decisions. As a result, tasks that fall within these areas yield accurate judgments which, in some cases, rival the performance of more complex, normative models, whereas tasks that fall outside of these areas (both in terms of what information is easily focused on and what judgments the processes are designed to produce) will be more difficult and prone to error – precisely because they fall beyond these domains.

Mental Representation of Numerical Information

Once quantitative information moves from the environment to the mind, there is the issue of how it is represented in the mind. Several possibilities exist. For instance, if I look out over a classroom and pay attention to the number of male and female students, that information could be represented as frequencies (15 students are male and 30 students are female), as percentages (33% of the students are male and 66% of the students are female), as a fraction (1/3 of the students are male and 2/3 of the students are female), as a ratio (there is a 1:2 ratio of male to female students), or as single-event probabilities (a student has a 0.33 probability of being male, and a 0.66 probability of being female). Each of these formats has slightly different mathematical properties, and each has advantages and disadvantages (in many cases, these representational formats were invented specifically to provide certain advantageous properties in particular contexts). In theory all of these formats are also interchangeable, although certain conversions require supplemental information (e.g., given only that 33% of students are male, converting to the actual frequency of males requires also knowing the total number of students).

The heuristics and biases perspective does not generally view any of these formats as fundamentally different from the others. Formally equivalent statistical information is viewed as cognitively equivalent, whatever the format. This is, in fact, a perspective that is shared by reasoning theories such as the mental models approach (Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999). It is a relatively domain-general view of cognitive abilities (i.e., judgment, decision making, and reasoning processes are applied by design with equal effectiveness across a wide range of domains).

The ecological rationality approach has focused on frequencies as a differentially useful information format, in part because of the dominance of this format if one is considering the extraction of information from the natural environment (see previous section). Research has documented that frequency presentations of information lead to better judgments and decision making (Cosmides & Tooby, 1996; Gigerenzer, 1991; Gigerenzer & Hoffrage, 1995). While this initially might seem ominous for a heuristics and biases viewpoint, it has instead led to greater clarity and refinement in thinking. Much earlier, Tversky & Kahneman (1983) actually noted a similar incidental finding that frequencies led to improved performances over single-event probabilities. Their explanation for the effects of such "seemingly inconsequential cues," however, was that "explicit reference to the number of individual cases encourages subjects to set up a representation of the problems in which class inclusion is readily perceived and appreciated" (p. 309). Thus, this perspective views natural frequencies as fundamentally equivalent to single-event probabilities, with the difference in performance reflecting the individuals' cognitive representation of the former.

Closer inspection reveals that certain types of frequency representations inherently carry information about base rates that other representational formats do not provide (recall the example of needing the total number of students to convert a percentage to a frequency of male students in a classroom). If base rate information is needed to make an accurate judgment or decision, frequencies are easier to work with –not because they are mysteriously special, but rather because they conserve information about the base rates (Kleiter, 1994). Thus, determining the posterior probability of a hypothesis (a process that takes base rates into account) requires using the following form of Bayes theorem when statistical information is represented as single-event probabilities, percentages, or fractions:

$$p(H|D) = \frac{p(D|H)p(H)}{(\ [p(D|H)p(H)] + [p(D|{\sim}H)p({\sim}H)]\ )}$$

where D is the known Data and H is the Hypothesis of interest that is evaluated in light of the data. Since, however, natural frequencies entail the sample and effect sizes, judgments that derive from these formats require a much simpler equation: Rather than computing the above conditional probabilities and combining these quantities with the base rates, the respondent need only attend to the joint probability of the data in the presence versus the absence of the hypothesis. Stated more formally, since, according to the conditional probability rule, $p(D|H)\ p(H) = p(D \cap H)$ and $p(D|{\sim}H)\ p({\sim}H) = p(D \cap {\sim}H)$, it follows (from the above theorem and the equivalence relations) that:

$$p(H|D) = \frac{p(D \cap H)}{p(D \cap H) + p(D \cap {\sim}H)}$$

Both theories recognize that the use of frequencies (specifically, naturally sampled frequencies) simplify the computational complexity of probabilistic reasoning (as explained from a frequentist perspective by Gigerenzer & Hoffrage, 1995). This is often described by the heuristics and biases program as making the structure of the problem more transparent, or clearer to people, but in fact it is fundamentally about the nature of the information provided – not mental clarity. In other words, as both views agree that Bayes theorem is computationally less complex under natural frequency rather than single-event probability formats, the issue of disagreement is not whether natural frequency formats are clearer than percentages, but why. The nature of the information provided is important to the extent that it informs the issue concerning the cognitive processes that underlie intuitive probability judgment. This "frequency effect" has generated considerably more controversy than one would expect if it was a simple matter of easier math. More must be at stake to lead Johnson-Laird, et al. (1999, p. 81) to claim "It is indeed crucial to show that the difference between frequencies and probabilities transcends mere difficulties in calculation." The crucial element is that if the representation and processing of numerical information is determined, in part, by the statistical format in which the environment presents this information, it follows that this ecological factor, and not limitations in processing capacity alone, may contribute to the observed pattern of intuitive probability judgments—thus bringing the accepted wisdom of the heuristics and biases view into question.

So it is crucial to determine if, as many with an ecological rationality view claim, frequencies are a privileged representational format in the human mind. Proponents of the ecological rationality view point out that frequency information, besides inherently carrying more information and being more flexible, is more ecologically valid (e.g., Brase, Cosmides, & Tooby, 1998). In the real world, outside of casinos, racetracks, and decision-making research labs, information tends to exist in frequencies (and, as discussed in the next section, frequencies within a system of natural sampling). The prevalence of frequency information in the real world, which has been an enduring property of the world for millennia, also indicates that evolved abilities for attending to statistical information about the world would likely be designed to accept frequency data.

As both the heuristics and biases and ecological rationality views agree that natural frequencies simplify the computational demands of many judgment and decision making tasks, it is important in evaluating these theories to identify whether the observed pattern of facilitation under natural frequencies is due to computational simplification alone, as the heuristics and biases view proposes, or whether the improved performance is a consequence of both computational simplicity and the mesh between natural frequencies and an evolutionary adaptation to process these formats, as the ecological rationality view advocates. Distinguishing between these two possibilities is difficult, because the tasks traditionally used in this research involve exactly the sorts of computations that get simpler with the use of frequencies. Brase (2002b), however, found that when people are asked to evaluate the clarity and understandability of statistical statements (i.e., with no computational confounds), frequencies are seen as clearer and easier to understand than single-event probabilities. Additional lines of evidence come from work in developmental psychology. Infants appear to be predisposed to attend to and use frequencies (i.e., natural, whole numbers), and effects of the privileged status of frequency representations can be observed later in life in mathematics education, as fractions and decimal notations are chronically misconstrued as frequency notations by children (Brase, 2002c). It has even been found that judgments under uncertainty involving items that are difficult to conceptually parse into appropriate parts for counting (i.e., frequencies) produce poorer judgment performances (Brase, et al., 1998). The fact that individuation into parts that are perceived as discrete elements facilitates statistical reasoning about those parts strongly indicates that the cognitive mechanisms involved are designed to take discrete, countable (i.e., frequency) information as inputs.

Against mounting evidence that the facilitory effects of natural frequencies are due to more than just computational simplification, heuristics and biases supporters have begun to take the offensive. Girotto & Gonzalez (2001), for instance, have asserted that people actually can be good at complex judgments and decisions (e.g., infering posterior probabilities) when given only probabilities for unique events. These authors reasoned that if the "frequency effect" can be found without frequencies, then the theoretical backbone of this effect (and challenge to the heuristics and biases view of the mind) collapses. The probabilities used by Girotto & Gonzales (2000, 2001), however, are a specific type of "probabilities," which are stated in simple numerical terms, such as:

> Mary is tested now [for a disease]. Out of the entire 10 chances, Mary has ___ chances of showing the symptom [of the disease]; among these chances, ___ chances will be associated with the disease. (p. 274)

How many times was Mary tested? Once or ten times? If tested once, there is one "chance" for a result (about which we could discuss subjective confidences, but that is a different issue). If tested 10 times, then this is an example of frequency information. It seems quite suspicious to say that subjects are truly reasoning about unique events, not utilizing cognitive mechanisms designed for dealing with frequencies, when the probabilities are stated as *de facto* frequencies (i.e., 3 out of 10). Although Girotto and Gonzales (2001) claim that "chances" refer to the probability of a single-event, it can just as easily be argued that this format yields better reasoning because it manages –in the view of the research participants— to tap into a form of natural frequency representation.

The argument that these natural frequency-like "chances" are interpreted by research participants as natural frequencies is an empirical question, and current research (Brase, in prep) indicates that this truly is the case. Participants were given statistical word problems, based on those used by Girotto and Gonzales (2001). Some of these problems used the natural sampling-like chances wording:

- The applicants for admission to a prestigious university have to pass an entrance examination that involves both an oral test and a written test. Here is some information about the results of last year's examination.
- An applicant had 5 chances out of 100 of being accepted.
- 3 of the 5 chances of being accepted were associated with passing the oral test.
- However, 19 of the remaining 95 chances of being rejected were also associated with passing the oral test.
- Imagine Jean is an applicant taking the entrance examination.
- Out of the _____ chance(s) that Jean will pass the oral test, there are _____ chance(s) she will be accepted.

Other versions of this problem were given that used either percentages (not a natural sampling format) or used a non-"chances" wording that clearly provided a natural sampling format. After solving these problems, the participants were asked how they had thought about the information and reached their answer to the problem:

A) I thought about the information as a single application with some possibility of having been successful on the oral test and some possibility of having been accepted.
B) I thought about the information as a large number of applications, some of which were successful on the oral test, and some of which were accepted.
C) Other: I thought about the information as _________

Of interest is not only how many individuals in each wording condition gave the correct answer, but how many of those correct answers were derived from a frequentist interpretation of the content (i.e., responded "B" on the follow-up question). As Figure 1 clearly shows, the participants who interpreted the chances as frequencies were significantly more likely to reach the correct answer (no participants given the information in the percentage format were able to provide the correct answer). Participants who were given natural frequencies from the start were relatively likely to reach the correct answer regardless of their reported interpretation.

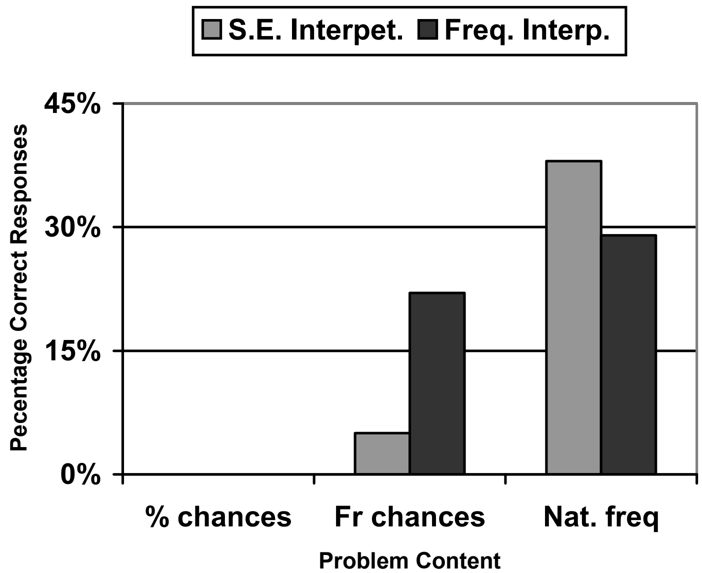

Figure 1: Partial results from Brase (in prep), showing that frequencies couched within "frequency-chances" phrasing in a Bayesian reasoning problem are more likely to produce the correct answer when interpreted as frequencies, rather than as single-event (S.E.) probabilities

It appears that one must be cautious about imputing a specific interpretation of stimuli to one's research participants when there is more than one possible interpretation they could be making. More specifically, numerical information labeled as "chances" can be mentally represented by participants as natural frequencies within some contexts, regardless of how they have been labeled in the experimental design. So the question of if numerical formats other than frequencies can produce the "frequency effect" is still quite open. Much of the confusion in this area has been due to conflations of experimenter's designations with the questionable attributions of mental representations to participants. (See Hertwig & Gigerenzer, 1999 for similar findings within a different context)

Processing Numerical Information

The debate over how numerical information is initially represented in the mind is compounded by related issues regarding how this information is subsequently organized and processed. The initial representational state of quantitative information has implications for what types of processing will be possible and how this information can be instantiated (e.g., binary numbers, Arabic numbers, and Roman numbers each require different procedures for making calculations, and allow different types of calculations). In many cases, including that of using base rates to calculate posterior probabilities, the initial numerical representation is just the beginning of the judgment or decision making process.

A variety of researchers have contested the claim that data in the form of frequencies (which presumably generates frequency mental representations) is a blanket panacea for effective and accurate processing of numerical information. The extreme form of this claim, however, is a straw-man argument; the fact that frequencies can facilitate statistical reasoning – even the assertion that frequencies are a cognitively privileged representational format—in no way precludes other factors from inhibiting statistical reasoning abilities. To take a facetious example, a judgment problem written in Sanskrit and presented to English-speaking subjects is probably no easier when frequencies are used as compared to other numerical formats.

The problem with the general criticism that frequencies do not always produce facilitation is that the claim is validated only by artificially separating frequencies from the natural sampling system in which they are usually found. That is, research in the ecological rationality approach actually involves one specific type of frequencies; those called "natural frequencies." A natural frequency is the result of the sequential acquisition of event frequencies from experience (i.e., natural sampling; see Aitchison & Dunsmore, 1975; Kleiter, 1994; Gigerenzer, 1998). This method of information gathering by counting events as one encounters them and embedding the counts in a categorical conceptual structure (e.g., see Figure below) produces frequencies that are not individually normalized to an arbitrary reference class (such as 100 for percentages). Instead, frequencies based on natural sampling –i.e., natural frequencies – implicitly carry information about the base rates via their relative sizes. Obviously, this is not a claim about frequencies in general, but a specific format of frequencies that have been chosen because they map on to the structure of statistical information as it is encountered in the natural world (or specifically in the environment in which human cognition evolved).

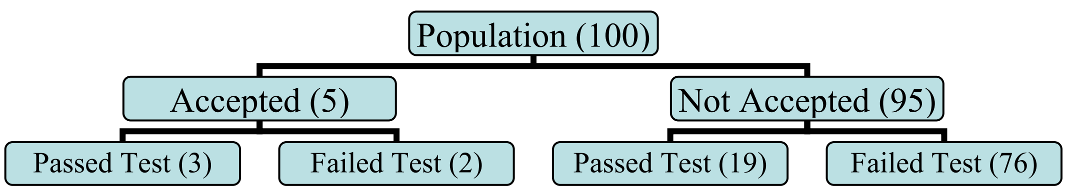

Figure 2: An example of a natural sampling framework, including frequencies (in parentheses) that depicts the university admissions problem (Girotto & Gonzales, 2001).

One of the key properties of natural frequencies (for present purposes) is that the categories of information are interrelated (e.g., nested within or subsuming other categories). For natural frequencies, this property is simply an automatic result of how the statistical information is obtained (via sequential acquisition and organization of event frequencies). Challenges to the ecological rationality viewpoint have repeatedly re-invented this key property as their own discovery and tried to hold it separate from "mere frequencies." For example, Johnson-Laird, et al. (1999) reintroduce the basic relevant principle of natural sampling as their "subset principle," even implying that ecological rationality researchers somehow missed this property (even though they cite work that discusses –and names— the principle at length):

> The real burden of the findings of Gigerenzer and Hoffrage, (1995) is that the mere use of frequencies does not constitute what they call a 'natural sample.' Whatever its provenance, as

they hint, a natural sample is one in which the subset relations can be used to infer the posterior probability, and so reasoners do not have to use Bayes's theorem. (p. 81).

More broadly, this is most likely the aspect to which Tverky and Kahneman (1983) were referring when they speculated that frequencies helped people represent class inclusion. A number of researchers have "discovered" this aspect of natural sampling and given it a new label, including:

1. Johnson-Laird and colleagues (Johnson-Laird, et al., 1999; Girotto and Gonzales, 2001) discussed the "subset principle," which is actually an abstraction of the natural sampling structure;
2. Evans, Handley, Perham, Over, and Thompson (2000) proposed a process that involves "cueing of a set inclusion mental model," rather than a natural sampling structure;
3. Macchi (1995) and Macchi and Mosconi (1998) have created the label of "partitive formulation" to describe the natural sampling structure; and
4. Sloman, Over, Slovak, and Stibel (2003) use the term "nested-set relations" rather than natural sampling, following Tversky and Kahneman (1983).

In each of these cases the discovered principle, process, formulation, or relation is, in fact, a natural sampling framework, the conception of which predates all these (see also, Hoffrage, Gigerenzer, Krauss, & Martignon, 2002).

Girotto and Gonzales (2000) provide a nice example of how one can separate the representation of information in frequencies from a system of natural sampling:

According to a recent epidemiological survey:

- Out of 100 tested people, there are 10 infected people.
- Out of 100 infected people, 90 people have a positive reaction to the test.
- Out of 100 non-infected people, 30 have a positive reaction to the test.
- Imagine that the test is given to a new group of people. Among those who have a positive reaction, how many will actually have the disease? ____ out of ____.

The "mere use" of frequencies here does *not* constitute a natural sample and that is an important part of the problem's difficulty. The solution to this problem requires using the traditional form of Bayes theorem, which requires converting the numbers into single-event probabilities and then combining these quantities with the base rates, which means this problem is actually *more* difficult to process than a problem expressing data in terms of probabilities, since only the former requires this initial conversion (recall the extreme version of the criticism that frequencies per se did not guarantee correct statistical reasoning). Presenting frequencies outside of the context of a natural sampling system (i.e., as "mere" or more specifically, normalized, frequencies) is therefore an unnatural situation.

Two more advantages of a natural sampling system are that it conserves valuable information and is more flexible, as compared to formats such as percentages, fractions, and single-event probabilities. To illustrate the first of these points, consider the above problem used by Girotto and Gonzales. From a natural sampling perspective, as well as in other ways, this frequency representation is a degenerative case, since, for example, it fails to conserve information about the sample size of the reference class and to partition the relevant quantities into natural samples. If one changes the frequencies to fit with a natural sampling system, however (and clears up semantic ambiguities and phrasing inconsistencies), the problem becomes clearer:

- According to a recent epidemiological survey on a particular disease, 10 out of every 100 people have the disease.
- A test exists to detect this disease, but this test is not perfect. It does not always detect when a person has the disease, and at other times the test indicates that a perfectly healthy person has the disease (called a "false positive").
- Specifically, only 9 out of every 10 people who have the disease get a positive result from the test for this disease.
- Additionally, 27 out of every 90 people who do not have the disease also get a positive result from the test for this disease (false positives).
- Suppose the test for this disease is given to a random sample of 100 people.
- How many people, out of those who have a positive result on the test, will actually have the disease? ____ out of ____

Instead of using totally new reference classes in each of the sentences, the problem now has one reference class of 100 people tested, with various subsets identified. This is the sort of natural sampling situation one is likely to encounter in the real world (e.g., you have 100 friends, who can be segregated into various subtypes…). It should be no surprise that when Girotto and Gonzales (2000) gave their original version of this task to participants, literally *none* of them were able to obtain the correct answer. This revised version of this problem, however, yielded significantly better performance (Brase, 2002a). From an ecological rationality perspective, there are several reasons to expect better performance on the revised problem:

1. the natural sampling structure corresponds to the way that information is typically encountered in the real environment,
2. (also as a consequence of the natural sampling structure) the computational requirements for solving the problem are less severe, and
3. the problem is more clearly stated, including a consistent syntax for both presenting data and answering the problem.

Clearly, the conclusions of Girotto & Gonzalez (2000) overstate the implications of their results, especially results with 0% correct responses that may suffer from undeterminable floor effects.

Information about the *natural* frequencies of different events within this population is conserved in natural sampling, and this leads to another advantage of a natural sampling system, its flexibility. Because natural sampling tracks the simple frequency of occurrences, independent of any other events, one can in theory start from any event and calculate the conditional probability of any other event. (see Brase, 2001a for further elaboration on this point.)

There is a very real issue within all of these parallel terminologies for natural sampling, and that is the question of whether it is possible to represent natural sampling structures (or nested set relations, etc.) using information that is not frequentist (i.e., that is not either explicitly frequencies or information that is quite easily construed as frequencies). The conundrum is that in order to create such a structure (i.e. with “sets” or “parts”, etc.) one must have discrete units with which to work. This directly implies that these discrete units will be countable entities, and therefore frequencies. It appears, thus far, that natural sampling structures are inextricably linked with frequency representations.

Evaluating Rationality

It is broadly agreed that, aside from issues of how to characterize numerical formats and how to label natural sampling frameworks, certain manipulations can modulate the apparent competence of human statistical reasoning. These performance changes are not merely compared to each other; they are also held up against certain independent criteria. The extent to which people perform well or perform poorly is often decided by comparison to a criterion of what is “good” or “correct” performance, and meeting that criterion is, roughly, what’s called “rationality.” This yardstick of rationality is an important evaluation of human nature in general. The controversy emerges, at this level, because there are different possible yardsticks of what constitutes the appropriate model of human rationality. A single, normative model of rationality is favored by rationality pessimists, whereas a pluralistic, ecological model of rationality is favored by rationality optimists.

The normative rationality of the heuristics and biases approach is justified by at least two points. First, there is the fact that this approach derives much of it’s heritage from the field of economics, in which there is a long and fruitful precedent for using single and broadly applicable criteria for making judgments and decisions (e.g., the concept of utility, including expected utility and subjective expected utility). Although the heuristics and biases view largely rejects the normative models of economics, and appeals instead to non-extensional models of subjective probability (e.g., Support Theory), this view nonetheless assumes that the elements of a situation should be reducible to some common metric, which can then be used to determine the best course of action. The second justification for this approach is that a single criterion for rationality is more parsimonious and avoids problems associated with positing multiple –and therefore potentially conflicting – criteria for rationality across different domains.

The ecological rationality approach accepts multiple norms for human performance across different contexts, and supports this position by emphasizing the criteria of mutual consistency and integration across multiple scientific fields. The single rationality benchmark often assumed in psychology is, in fact, just one plank within a larger system of rationality criteria utilized across many fields. For example, the field of logic is not comprised of just basic Aristotelian logic, but instead emphasizes that different forms of logic (e.g., predicate, modal, deontic, etc.) apply in different circumstances. Often these non-classical logics are inconsistent with Aristotelian logic (e.g., there exist sequents that are valid under the former and not the latter, and vice versa), so it remains unclear how –or if— these non-classical logics could be integrated within a unified, coherent theory of logic. Rational evaluations of probabilistic information, according to the field of probability theory, can shift depending on if that information is objectively derived information or subjective confidence estimates (e.g., Hertwig & Gigerenzer, 1999). If a common denominator for rationality exists within the ecological rationality approach, it is based not on formal logic or probability theory, but on the inclusive fitness consequences criteria of evolutionary biology. Even this criterion, however, is only an abstract consideration that has to be balanced against real-world and real-time limitations (e.g., Rhode, Cosmides, & Tooby, 1999).

Differences between these competing yardsticks of rationality are not only important in terms of understanding human nature in general, but they can also have implications regarding the allowable changes to situations – changes made, for example, to improve performance. For instance, it has been the position of the ecological rationality camp that information in natural frequencies can greatly facilitate bayesian reasoning (Cosmides & Tooby, 1996; Gigerenzer & Hoffrage, 1995), but this assertion has been hotly contested by others as completely untrue (e.g., Howson & Urbach, 1993). Much of the difficulty here is due to the confounding of two possible meanings of "bayesian reasoning." One interpretation – apparently used by Howson & Urbach – is that "bayesian reasoning" refers to the use of the classic formula for Bayes's theorem (the first formula given previously), in which case natural frequencies (not being in a probabilistic format) cannot be bayesian. A second interpretation, used by Cosmides & Tooby, is that "bayesian reasoning" refers to reasoning in a way that achieves the function that Bayes's theorem is designed to achieve (i.e., resulting in the calculation of a posterior probability consistent with Bayes' Law). Under this interpretation, natural frequencies can and do elicit bayesian reasoning. Furthermore, this reading accepts that reasoning which starts with natural frequency representations, manipulates those frequency representations, and produces frequency outputs, can nevertheless embody aspects of a calculus of probability. It is true that, if one is using natural frequencies and not converting them to some other format, the traditional form of Bayes's theorem is inappropriate, much like entering binary numbers into a regular calculator is inappropriate. The utility of Bayes's theorem in its conventional form is that it expresses a set of relationships between different probabilities that is agnostic as to whether they are derived from frequency information or single-event confidences. However, this utility is purchased at a cost of computational power and flexibility.

A final note can be made regarding rationality; there is another perspective on this issue that can be called "practical rationality"—the perspective that whatever approach manages to achieve useful results is the most rational approach to take (e.g., Evans, Over, & Manktelow, 1993). This is a much less theoretically driven approach, and although it tends to favor the ecological rationality approach by it's focus on the fit between behavior and the environment, it is in principle affiliated with neither the ecological rationality nor the heuristics and biases views . In the relatively short span of the past 10 years, the ecological rationality approach has made significant inroads in terms of demonstrating the practical rationality of adopting this view. Specifically, the principle of using natural frequencies to facilitate better understanding of quantitative information has been readily applied to various real-world uses. Research has demonstrated that it can be used to improve educational objectives of teaching statistical inference (Ruscio, 2003; Sedlmeier & Gigerenzer, 2001), to improve the assessment of statistical information by courtroom judges (Lindsey et al., 2003), and to improve the communication and understanding of medical statistics by physicians and patients (Gigerenzer, Hoffrage, & Ebert, 1998; Hoffrage & Gigerenzer, 1998; Hoffrage, Lindsey, Hertwig, & Gigerenzer, 2000).

CONCILIATION

Yet another truism is that, after two opposing viewpoints interact for a while, an emerging consensus is some combination of the two views rather than a winner-takes-all result. *Dual process models* offer just such an intermediate position. As the label indicates, dual process models propose that cognitive processes can very generally be divided into two systems (Evans & Over, 1996; Stanovich & West, 2000; Sloman, 1996). These two systems have been given a number of labels by various authors, but have lately been generalized as "System 1" and "System 2" (see Stanovich & West, 2000). System 1 processes are relatively automatic, largely unconscious, and less demanding of computational capacity; it has variously been called a heuristic, implicit, intuitive, and experiential system. System 2 processes are relatively effortful, requiring more conscious deliberation, and demanding of cognitive attention; it has been called the analytic, explicit, rule-based, and rational system. There are indications that the dual process model may be – if not a resolution – at least a meeting space for the heuristics and biases approach and the ecological rationality approach. This is a significant conciliation that represents meaningful accommodations by both approaches.

A More Subtle Heuristics and Biases Approach

The traditional approach in the heuristics and biases research tended to be of the view that, in dual process model terms, people tried to use System 2 processes but were unable to do so successfully and therefore fell back on System 1 processes. For example, Tversky and Kahneman (1982, p. 19) point out:

> "A person could conceivably learn whether his judgments are externally calibrated by keeping a tally of the proportion of event that actually occur among those to which he assigns the same probability. However, it is not natural to group events by their judged probability. In the absence of such grouping it is impossible for an individual to discover, for example, that only 50 percent of the predictions to which he has assigned a probability of .9 or higher actually come true."

More recently (e.g., Kahneman & Fredrick, 2002), the heuristics and biases view has adopted a dual process account, according to which judgments are *initially* and by default processed by fast, associative (System 1) mechanisms, that reflect a range of errors and biases (i.e., heuristic processes). The output of this decision process may then be evaluated by a more deliberative, possibly rule-based, system (System 2) that either accepts or rejects the heuristic response. In other words, the sequence utilization is reversed from earlier ideas. Two consequences of this shift, which brings the heuristics and biases account more closely in line with other dual process model accounts, are that heuristic processes can operate first and by default, and that it may be the case that heuristics processes are utilized even when the computational demands of the task do not exceed the processing limitations of the mind.

The question of which system is invoked first by judgment processes is important because it holds implications for both the effects of motivation on performance and the assessments of human rationality. Increasing motivation will increase performance only if the

System 2 capabilities are initially not fully tapped, rather than initially overwhelmed. Similarly, prospects for human rationality look somewhat more hopeful if System 2 processes are secondary (but intact) rather than primary processes that are overwhelmed. The shifting of System 1 processes to the forefront in judgment processes, however, raises philosophical issues: If the System 2 processes are the normatively correct (analytic, rule-based, normatively rational) processes, why do they take a back seat to System 1 processes? Is this, in and of itself, a form of irrationality? What do these System 1 processes have that make them so deserving of being on the front-line of human cognition? Furthermore, what evolutionary and ecological factors led to the development and utilization of System 1 and System 2 processes according to the heuristics and biases view?

In summary, the empirical evidence has led the latest heuristics and biases approach to place System 1 processes in front of System 2 processes, which creates a greater sense of importance and power to System 1 processes, but it is not theoretically clear why this should be the correct placement.

A More Subtle Ecological Rationality Approach

The ecological rationality approach, in contrast, has tended to focus on what, in dual process model terms, would be considered System 1 processes and paying much less attention to System 2 processes. A large part of this contrasting focus may be due to the fact that the ecological rationality approach has fairly strong and clear theoretical concerns that place System 1 processes in the forefront of judgment processes. Evolutionary theory indicates that the mind should be constructed of adaptations that produce behaviours that have been adaptive in a species' evolutionary past. Furthermore, these adaptations will be biologically specified to the extent that aspects of the environment, other associated cognitive systems, and the natural selection process are stable and allow such specification. The result is System 1 processes that are relatively fast, experientially easy or automatic (which is not to say that they are necessarily simple in strict computational terms), and less open to introspection. These processes will also be well adapted to function in the environment in which the species in question evolved.

On the other hand, System 2 processes are much less customized to the evolutionary and ecological circumstances of a particular species, and they are not as fast, easy, automatic, or unconscious. System 2 processes are much more likely to rely on purposeful construction, such as formal education, in order to develop beyond rudimentary capabilities.

It is now a bit clearer as to what, exactly, the ecological rationality view is "optimistic" about – the usefulness of System 1 processes. To say that System 1 processes are quite useful, however, should not be to derogate System 2 processes. There does seem to be room within the ecological rationality perspective for a dual process framework. Gigerenzer & Regier (1996), speaking from an ecological rationality perspective, are open to a dual process framework but also stress the need for better development of the nature of these systems and how these systems relate to one another.

Issues Outstanding

From either perspective, clarification of the nature and function of a dual-process model is imperative. It is even becoming clear that the term "dual-process" is a misnomer, as both systems appear to be composed of collections of processes. One area that may help with this clarification –as well as being a topic of some interest and controversy in it's own right—is the question of how best to characterize the domains of evolutionary adaptation that are supposed to underlie different System 1 processes. Evolved human psychological adaptations exist, just as surely as evolved physical adaptations such as lungs, bipedalism, and eyes exist, but exact definitions have thus far been contentious (Bock & Cardew, 1997). Indeed, this general issue of defining domains is one that subsumes more specific topics such as whether numerical information is processed in terms of nested sets or in terms of natural sampling; the more abstract idea of nested sets constitutes a broader, more general domain of application (and hence a more general evolutionary adaptation) than the idea of natural sampling.

This issue of the boundaries and specificity of domains in which judgment and decision-making (System 1) mechanisms operate has sometimes been confused with other issues. This issue is not about "evolutionary" versus "non-evolutionary" explanations (or similar divisions between biological/environmental, genetic/cultural, or natural/learned). These are all artificial dissociations; the functional structure of the mind is never totally independent from evolutionary concerns (nor are they ever totally independent of environmental concerns). To posit such a distinction usually involves perpetuating a false nature/nurture dichotomy.

There is a real issue, however, in the questions of what a cognitive mechanism was designed to do (it's *proper* function, in Sperber's terms, 1994) and how far outside of those specifications that mechanism can continue to function effectively (Sperber's *actual* function). The former will, of course, have very real consequences for the latter. A mechanism designed to perform a very specific proper function, within a very narrow domain, will be likely to have a relatively restricted potential for actual functionality. Whether a mechanism is relatively specific or general in it's functionality, however, it has in either case been designed to some extent by evolution. For example, Boster (2002) has found that the mechanism that is thought to underlie biological similarity judgments based on perception (e.g., recognizing different types of birds) has a fairly general scope of actual functionality (e.g., it can generalize to abstract perceptual classes, such as Fourier blobs). The correct conclusion made from this work was not that it was a "non-evolutionary" mechanism, but rather that "evolution may craft solutions to specific problems using general mechanisms" (Boster, 2002).

From this perspective, there are a number of potential situations that could be accurate descriptions of human cognitive architecture in this area:

a) It could be the case that the proper domain of the cognitive mechanism being used is one of natural sampling of frequencies, and not extending beyond that to a more extensive actual domain.
b) It could be the case that the proper domain of the cognitive mechanism being used is one of natural sampling of frequencies, but with an actual domain that extends more widely (but not extending to cover the entirety of all forms of nested sets).

c) It could be the case that the proper domain of the cognitive mechanism being used is one of natural sampling of frequencies, but with an actual domain that extends to effectively cover all other forms of nested sets.
d) It could be the case that the proper domain of the cognitive mechanism being used is that of all nested sets, in which case the actual domain would likely be isomorphic.

In principle, there are ways to discriminate between these options (e.g., see Andrews, Gangestad, & Matthews, 2002), but in practice it has so far been quite difficult. One difficulty is that it is not apparent how to evaluate a natural sampling or nested set framework without utilizing content that is discrete, countable information (i.e., frequency information, even if the information is frequencies only in how it is perceived). In other words, since natural frequencies entail nested-set relations and because computing Bayesian inferences under natural frequencies entails elementary set operations, it may not be possible to experimentally differentiate these views.

Resolving issues such as this may come down to converging lines of evidence from other research areas. Does the mind embody domain specific (System 1) inductive reasoning mechanisms that encode statistical regularities in the environment as natural frequencies? Alternatively, do such inductive reasoning mechanisms reflect elementary set operations that derive from the rule-governed processes of System 2, as the heuristics and biases view advocates? Can neurophysiological correlates be found for these particular cognitive processes? Furthermore, do comparative studies of other species reveal similar inductive reasoning competencies that recruit an analogous pattern of brain activity? So in the end, both pessimists and optimists will have to learn new ways of approaching the issue of mental representation of statistical information in order to continue the gradual convergence towards a true description of human cognition.

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